

Imaging Analysis of Photorefractive Phased Array Beamforming

Gregory Kriehn, Andrew Kiruluta, Paulo E. X. Silveira,
Sam Weaver, and Kelvin Wagner

Optoelectronic Computing Systems Center
Department of Electrical and Computer Engineering
University of Colorado, Boulder, CO 80309
Gregory.Kriehn@colorado.edu

ABSTRACT

We present the analytical description of a photorefractive phased array beamforming system using the BEAMTAP (Broadband and Efficient Adaptive Method for True-Time-Delay Array Processing) algorithm for a large N-element array that requires only 2 tapped delay lines (TDLs) instead of the conventional N TDLs. Simulation results indicate that the processor is able to adapt to a broadband signal of interest at a specific angle of arrival. We show that the system produces a coherent sum of the desired signals from the phased array, with the corresponding time delays appropriately compensated for in an adaptive fashion without prior knowledge of the angle-of-arrival.

Keywords: Adaptive phased arrays, photorefractive crystals, true time delay, optical processing.

1 Introduction

In this paper we provide an analysis of the optical implementation of a new approach to the problem of true-time-delay (TTD) beamforming for a large N-element array that needs only 2 tapped delay lines (TDLs) instead of the conventional N TDLs. The optical BEAMTAP system readily scales to¹⁻⁴ process signals from very large phased array radar systems with thousands of receiving elements with GHz of bandwidth. In contrast, earlier optical systems were restricted to at most 32 elements due to limitations inherent in multichannel Bragg cells.⁵⁻⁸ We present a simplified analysis of the TTD beamforming operation in an optical BEAMTAP system when the photorefractive crystal is in the image plane and diffraction is neglected, allowing for the theoretical analysis to remain simple while providing the greatest insight into the the algorithmic operation of the coherent optical processor while including appropriate carrier and bias terms.

2 BEAMTAP Imaging Analysis

2.1 System Overview

Figure 1 shows the architecture for the optical implementation of the imaging BEAMTAP system. A low relative intensity noise (RIN) laser is split by a beamsplitter (reflectance b_1) and frequency shifted by a modulator to within 1 GHz of the RF center frequency (typically ranging between 1 - 20 GHz). This beam is then distributed through a fiber feed network and is modulated by electro-optic modulators with the signal $s(t)$ coming from the phased array antenna. The effect of the first EOM is to act as a mixer or down converter, allowing for processing of large bandwidth signals which can be tuned over the full 1 - 20GHz RF band.² The modulated signals from the fiber feed network are then imaged into the photorefractive crystal using the 4f system L2.

The undeflected beam (transmittance $b_0 = \sqrt{1 - b_1^2}$) from the laser source is used as the input beam for the acousto-optic Bragg cell, or AOD. The beam is expanded and tilted to the Bragg angle, whereby the reference signal $r(t)$, applied to the AOD, launches a propagating acoustic wave which modulates the incoming light via the acousto-optic effect and diffracts it at the Bragg angle. Tangential matching in an anisotropic polarization switching device is used so that the diffracted beam has an orthogonal polarization to that of the undiffracted beam. Both of these beams are then imaged into the photorefractive crystal using the 4f lens system L1. The diffracted beam from the AOD interferes with the spatio-temporally modulated optical field launched by the fiber feed; and the time integration of this interference pattern produces 45° grating-lets wherever the beam containing the reference signal $r(t)$ is well correlated with one of the beams from the fiber feed network containing the signal of interest $s(t)$. The undiffracted light from the AOD does not write a grating since it has an orthogonal polarization to that of the phased array beam and is at the wrong frequency.

Light from the fiber feed network then diffracts off the grating-lets in the crystal. The time delays (both due to the inherent delays produced by the phased array antenna, and random time delays introduced by the small length mismatches between the fibers) are transformed into diffracted beams at spatial positions proportional to the time delays. These position dependent time delays will be eliminated by detection by a time-integrating traveling wave detector array appropriately velocity matched to the image of the AOD. Read-write multiplexing using angle and polarization encoding is implemented using a Rochon prism, polarizer, and diffractive optical element (DOE) in order to isolate the diffracted readout beam of the phased array from the diffracted write beam of the AOD. Details of read-write multiplexing architectures have been covered in previous publications and are only briefly delineated in this paper.^{4,9}

The diffracted signals are then imaged onto a time integrating traveling wave detector implemented with a time-delay-and-integrate charge coupled device (TDI CCD) where they interfere with the undiffracted light from the AOD. At each resolvable temporal increment, the scrolling TDI CCD is read out after the corresponding charge packet has traversed the length of the detector, accumulating and delaying detected signal contributions from each location. The resulting

Optical Implementation of True Time Delay Beamsteering

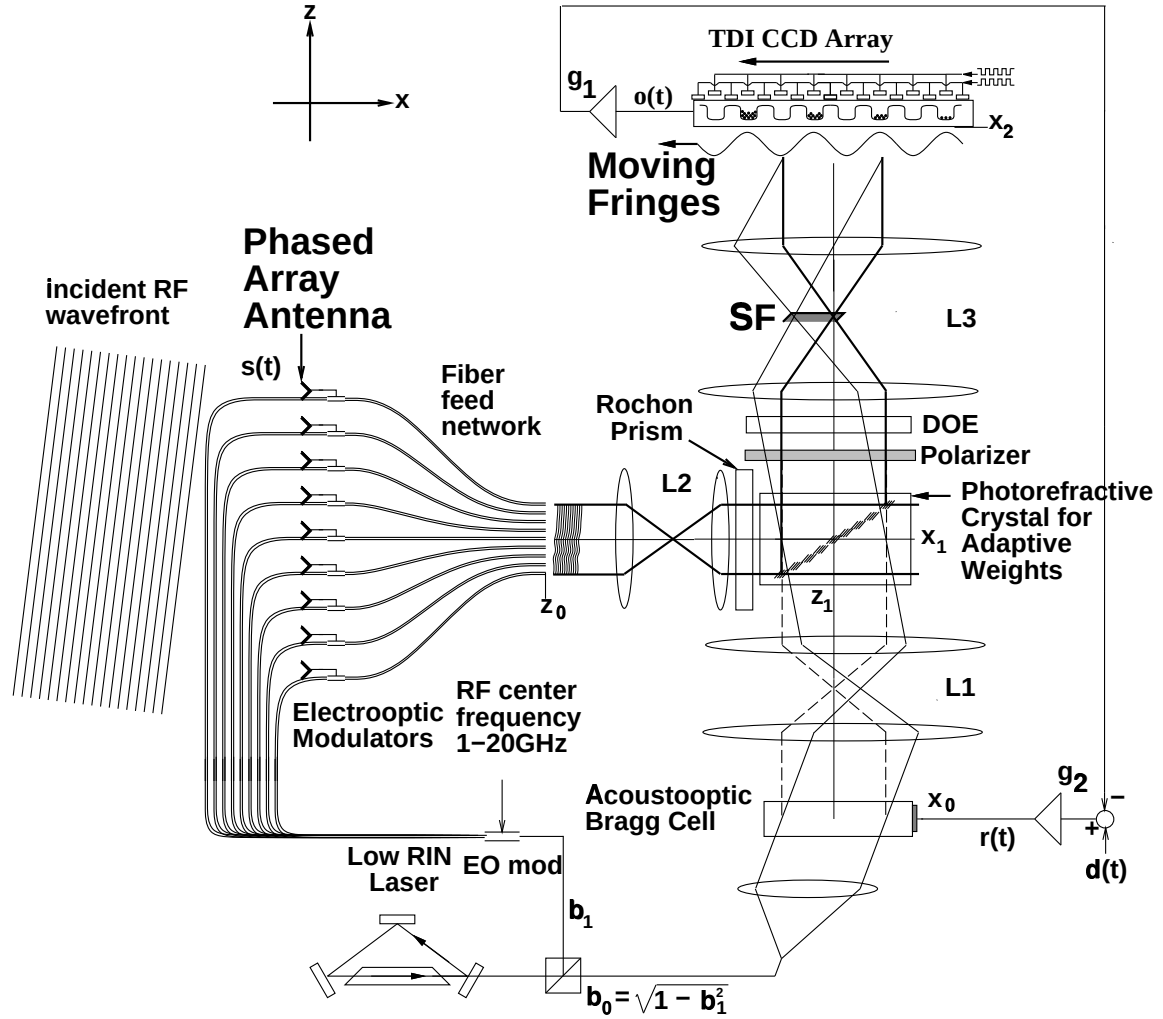


Figure 1: Optical implementation of BEAMTAP. Fiber feed network from phased array is shown on the left. Single coherent laser is split to drive both the fiber feed and the BEAMTAP processor. AOD interferes with signals from the array to form gratings in the photorefractive crystal. Diffraction of the phased array signals off of this grating is detected by a synchronous TDI CCD. The illustrated system places the photorefractive crystal in the image plane of the fiber feed and the AO, which are orthogonally propagating, in order to simplify the analysis presented in this paper.

output waveform is an adaptively weighted and coherently reconstructed version of the signal.² For closed loop adaptive processing, this output signal $o(t)$ is then subtracted from our desired signal $d(t)$, which is well correlated to the frequency spectrum of our signal of interest $s(t)$. This produces our error, or reference signal $r(t)$, which is then fed back into the AOD. The feedback

loop provides the necessary adaptation required to null out any undesired jammers present in the signal environment at the input of the phased array antenna—which can include both near-field and far-field jammers. In this paper, we analyze the open loop beam forming operation.

2.2 The AOD

If we consider an input RF signal to an AOD $r(t) = \tilde{r}(t)e^{j\omega_{rf}t}$ whose envelope $\tilde{r}(t)$ has been modulated at an RF frequency ω_{rf} , the signal will slide through the AOD at the acoustic velocity v giving a transmission

$$r(x_0, t) = \tilde{r}\left(t - t_r + \frac{x_0}{v}\right) e^{j[\omega_{rf}(t - t_r + \frac{x_0}{v})]}, \quad (1)$$

where t_r is a constant reference delay which allows arbitrary timing of the reference signal and for the AOD aperture to be centered in the optical system.

An optical beam with a carrier frequency ω_l which is incident on the AOD at the Bragg angle $k_B = k_o \sin(\theta_B)$ will diffract off the acoustic wave to produce a diffracted and undiffracted beam

$$E_{AO}(x_0, t) = b_0 e^{-j(k_B x_0 - \omega_l t)} \left[\sqrt{1 - \eta_{AO}^2 |\tilde{r}|^2} \hat{p}_x + \eta_{AO} \tilde{r}\left(t - t_r + \frac{x_0}{v}\right) e^{j[\omega_{rf}(t - t_r + \frac{x_0}{v})]} \hat{p}_y \right], \quad (2)$$

where $b_0 = \sqrt{1 - b_1^2}$ is the amplitude transmittance of the light from the beamsplitter, η_{AO} is the diffraction efficiency of the AOD, and $|\tilde{r}(t - t_r + \frac{x_0}{v})|^2$ has been written as $|\tilde{r}|^2$. Anisotropic diffraction in the AOD results in the diffracted and undiffracted beams having orthogonal polarizations, $\hat{p}_y$ and $\hat{p}_x$, respectively.

Once the diffracted and undiffracted beams are imaged through the lens L_1 , and noting that $x_1 = \frac{x_0}{M_1}$ where M is the magnification of the lens system, the electric field distribution can be completely described within the photorefractive crystal. For the sake of simplicity, the effects of diffraction are neglected; it is also assumed that the image plane is located within the crystal itself. For this analysis, we neglect diffraction so the propagation of the light within the photorefractive crystal is simply accounted for by including a propagation term in the z -direction

$$E_{AO}(x_1, z_1, t) = e^{-j(k_B x_1 - \omega_l t)} \left[\sqrt{1 - \eta_{AO}^2 |\tilde{r}|^2} \hat{p}_x + \eta_{AO} \tilde{r}\left(t - t_r + \frac{x_1}{v}\right) e^{j[\omega_{rf}(t - t_r + \frac{x_1}{v})]} \hat{p}_y \right] e^{jkz_1}, \quad (3)$$

where $k = \frac{2\pi}{\lambda_o} n$.

2.3 The Phased Array

Next, consider the signal produced by a far-field plane wave arriving upon a linear equally spaced array at some angle θ . The signal at a particular element n will have a differential time

delay between adjacent elements $t_p = \frac{d}{c} \sin \theta$, where d is the spacing between the elements and c is the speed of light. The signal at the n^{th} element can thus be described as $s_n = s(t - n t_p)$. Assuming that the phased array signal envelope $\tilde{s}(t)$ is modulated by a carrier ω_p , the signal detected by the n^{th} array element is

$$s_n(t) = \tilde{s}(t - n t_p) e^{j[\omega_p(t - n t_p)]} + cc, \quad (4)$$

More generally the nt_p term can instead be an arbitrary function of n ($f(n)$) to represent any random time delays caused by positional errors in the placement of the array elements, conformal arrays, or near-field signals.

Modulation by the first frequency shifting EOM offsets the light carrier by ω_s giving a new carrier $\omega_c = \omega_l + \omega_s$ (where we have ignored the other sidebands), and after being modulated at the fiber feed network, the electric field distribution launched by the fibers is

$$E_p(z_0, t) = \frac{b_1}{\sqrt{N}} \sum_{n=1}^N e^{j[\omega_c(t - t_n)]} a(z_0 - n D_0) \times \left[\sqrt{1 - \eta_p^2 |\tilde{s}|^2} + \eta_p \tilde{s}(t - n t_p - t_n) e^{j[\omega_p(t - n t_p - t_n)]} + cc \right] \hat{p}_y, \quad (5)$$

where $\frac{b_1}{\sqrt{N}}$ is the amplitude of the light at each of the fibers, N is the number of array elements, $a(z_0 - n D_0)$ is the aperture function of the n^{th} fiber at position $n D_0$, η_p is the modulation efficiency of the EOM's (assumed to be the same for all modulators), $|\tilde{s}|^2 \equiv |\tilde{s}(t - n t_p - t_n)|^2$, D_0 is the spacing between the fibers, and t_n is a random time delay which has been introduced to account for phase and time shifts due to the fiber feeds not being cut to exactly the same length—which affects both the light carrier ω_c and the phased array RF signal at ω_p . The polarization preserving fiber feed produces an output having the same polarization as the diffracted beam from the AOD, $\hat{p}_y$. The EOM is being operated as an amplitude modulator, but for the sake of simplicity the complex conjugate term will be subsequently dropped to produce a single-sideband phase modulator; including the conjugate term in the analysis would yield essentially the same overall final result.

This phased array signal beam is then imaged through L_2 , and noting that $z_1 = \frac{z_0}{M_2}$, $D_1 = \frac{D_0}{M_2}$, the electric field distribution within the photorefractive crystal can be described by including a propagation term in the x-direction

$$E_p(x_1, z_1, t) = \frac{b_1}{\sqrt{N}} \sum_{n=1}^N e^{j[\omega_c(t - t_n)]} a(z_1 - n D_1) \times \left[\sqrt{1 - \eta_p^2 |\tilde{s}|^2} + \eta_p \tilde{s}(t - n t_p - t_n) e^{j[\omega_p(t - n t_p - t_n)]} \right] e^{jkx_1} \hat{p}_y. \quad (6)$$

We will choose the carrier frequencies so that $\omega_l + \omega_{rf} = \omega_c + \omega_p = \omega_l + \omega_s + \omega_p$.

2.4 Formation of the Grating

As the optical beam from the AOD interacts with the beam from the phased array, an index grating is formed within the photorefractive crystal. Using first order analysis, the grating evolution can be described by

$$G(x_1, z_1, t') = \int_{-\infty}^{t'} E_{AO}(x_1, z_1, t) E_p^*(x_1, z_1, t) e^{-\frac{(t'-t)}{\tau}} dt + cc, \quad (7)$$

where the details of the photorefractive dynamics have been idealized.

For times $t' \gg \tau$ the convolution with $e^{-\frac{t'}{\tau}}$ results in a causal Lorentzian lowpass filter in the temporal frequency domain with a width $\Delta f = \frac{1}{2\pi\tau}$. Thus only stationary gratings can be written in the photorefractive crystal, which are then weighted by the photorefractive time constant τ . The spatial frequency response of the photorefractive crystal allows only high spatial frequency gratings to be recorded, although DC terms do lead to grating erasure.¹⁰

The undiffracted beam from the AOD has an orthogonal polarization to the entire phased array beam E_p^* , so it will be unable to write a grating. Thus the photorefractive grating is given by

$$\begin{aligned} G(x_1, z_1, t') = & \int_0^{t'} \left[\left[b_0 \eta_{AO} \tilde{r}\left(t - t_r + \frac{x_1}{v}\right) e^{j[\omega_{rf}(t-t_r)]} \right] \times \right. \\ & \left[\frac{b_1}{\sqrt{N}} \sum_{n=1}^N a^*(z_1 - nD_1) \left(\sqrt{1 - \eta_p^2 |\tilde{s}|^2} + \eta_p \tilde{s}^*(t - n t_p - t_n) e^{-j[\omega_p(t-n t_p-t_n)]} \right) \right] \times \\ & \left. e^{jk_o(z_1-x_1)} e^{j\omega_l t_n} \right] e^{-\frac{(t'-t)}{\tau}} dt. \end{aligned} \quad (8)$$

The diffracted beam from the AOD will try to write a grating with both the modulated and un-modulated beams from the phased array. Appropriate choice of the offset frequencies ($\omega_s + \omega_p = \omega_{rf} \equiv \omega_o$) allows only the modulated phased array beam to produce a stationary grating. The un-modulated EOM beam produces a moving grating with the diffracted AOD beam at the RF carrier frequency ω_o . Due to the lowpass nature of the photorefractive crystal this grating washes out.

$$\begin{aligned} G(x_1, z_1, t') = & e^{jk_o(z_1-x_1)} b_0 \frac{b_1}{\sqrt{N}} \eta_{AO} \eta_p \sum_{n=1}^N e^{j[\omega_o(t_n-t_r+n t_p)]} e^{j\omega_l t_n} a^*(z_1 - nD_1) \times \\ & \int_0^{t'} \tilde{r}\left(t - t_r + \frac{x_1}{v}\right) \tilde{s}^*(t - n t_p - t_n) e^{-\frac{(t'-t)}{\tau}} dt. \end{aligned} \quad (9)$$

Thus a cross-correlation is formed between the reference signal $\tilde{r}(t - t_r + \frac{x_1}{v})$ and $\tilde{s}(t - n t_p - t_n)$, the signals from each element of the phased array. The cross-correlation envelope is written as a modulation of the 45° gratings as indicated by the $e^{jk_o(z_1 - x_1)}$ term wherever $\tilde{r}(t - t_r + \frac{x_1}{v})$ overlaps with $\tilde{s}^*(t - n t_p - t_n)$ within the crystal. The grating-lets are written with a phase factor $e^{j[\omega_o(t_n - t_r + n t_p)]} e^{j\omega_l t_n}$ that accounts for optical propagation delays and RF signal delays at positions corresponding to their respective optical and RF time delays. Assuming these time delay distortions are small (less than 1/BW), the 45° grating-lets will lie on a tilted line, corresponding to a specific angle of arrival. Overall time delays can be accounted for by shifts of this line and changes in the angle of arrival are manifest as changes of the tilt angle of the envelope of the gratings. The grating-lets evolve as a function of time, allowing the processor to adapt to changing RF signal environments.

2.5 Diffraction of the Phased Array Signal Beam off the Grating

Now consider diffracting off of the grating with the signal beam. Wherever the signal from the fiber feed is well correlated with the signal from the AOD, strong gratings will be written within the photorefractive crystal. As light from each of the phased array signal fibers propagates through the crystal picking up a phase factor of e^{jkx_1} , each beam of light interacts with the local gratings and diffracts off of it, accumulating an additional phase of $e^{j[k_o(L - z_1)]}$ as each of the diffracted beams propagate to the end of the crystal. As a result, the diffracted beam can be calculated by,

$$\begin{aligned}
E_d^p(x_1, t') &= \int_0^L E_p(x_1, z_1, t') G(x_1, z_1, t') e^{j[k_o(L - z_1)]} dz \\
&= \int_0^L \hat{p}_y \left[\frac{b_1}{\sqrt{N}} \sum_{n'=1}^N e^{j[\omega_c(t - t_{n'})]} a(z_1 - n' D_1) \times \right. \\
&\quad \left. \left[\sqrt{1 - \eta_p^2 |\tilde{s}|^2} + \eta_p \tilde{s}(t - n' t_p - t_{n'}) e^{j[\omega_p(t - n' t_p - t_{n'})]} \right] e^{jkx_1} \right] \times \\
&\quad \left[e^{jk_o(z_1 - x_1)} b_0 \frac{b_1}{\sqrt{N}} \eta_{AO} \eta_p \sum_{n=1}^N e^{j[\omega_o(t_n - t_r + n t_p)]} e^{j\omega_l t_n} a^*(z_1 - n D_1) \times \right. \\
&\quad \left. \int_0^{t'} \tilde{r}(t - t_r + \frac{x_1}{v}) \tilde{s}^*(t - n t_p - t_n) e^{-\frac{(t' - t)}{\tau}} dt \right] e^{j[k_o(L - z_1)]} dz_1. \quad (10)
\end{aligned}$$

Unfortunately, the signal strength received from the phased array will usually be small, so that the modulation depth of the EOM is typically $\ll 1\%$. Thus the DC term in E_p which diffracts off the grating (although with an incorrect phase front due to the fiber time delays) could be much stronger than the modulated term, creating a bias signal so strong that it would swamp out the signal of interest at the TDI CCD. In order to avoid this, we will assume that the

EOM's use a suppressed carrier modulation scheme (or that a novelty filter is present to remove the DC carrier¹¹). The DC term will therefore not be present in E_p .

Note that the phase accumulated by propagating in the positive x -direction $e^{jk_o x_1}$ is cancelled with the x dependence of the phase within the grating $e^{-jk_o x_1}$; and at each z the phase accumulated by propagation to the output after diffracting off the grating $e^{j[k_o(L-z_1)]}$ cancels with the z dependence of the grating $e^{jk_o z_1}$. Furthermore, the random phase delays present in the carrier $e^{-j\omega_c t_n}$ are also cancelled, as well as the random phase delays present within the signal itself $e^{-j[\omega_o(n t_p + t_n)]}$. As a result, each of the components of light which diffract off the grating from the fibers pick up the same overall phase $e^{jk_o L}$. Because the fiber profiles do not overlap and are spatially orthogonal to each other, $n = n'$ and the two sums collapse yielding

$$E_d^p(x_1, t') = e^{j\omega_c t'} e^{jk_o L} b_0 b_1^2 / N \eta_{AO} \eta_p^2 \int_0^L \sum_{n=1}^N |a(z_1 - n D_1)|^2 dz_1 \times \\ \tilde{s}(t' - n t_p - t_n) e^{j[\omega_o(t' - t_r)]} \int_0^{t'} \tilde{r}(t - t_r + \frac{x_1}{v}) \tilde{s}^*(t - n t_p - t_n) e^{-\frac{(t' - t)}{\tau}} dt \hat{p}_y. \quad (11)$$

We choose the reference signal to be well correlated to the phased array signal $\tilde{r}(t) \approx \tilde{s}(t - T)$, where T is a constant time delay. We define the autocorrelation function as

$$R_{ss}(\alpha) = \int_{-\infty}^{\infty} \tilde{s}(t) \tilde{s}^*(t - \alpha) dt \Rightarrow |S(f)|^2, \quad (12)$$

where $\Rightarrow$ denotes the Fourier transform operation. For long integration times the $e^{-\frac{(t-t')}{\tau}}$ term simply scales the stationary autocorrelation function by the photorefractive time constant τ

$$\int_0^{t'} \tilde{s}(t + \frac{x_1}{v} - t_r - T) \tilde{s}^*(t - n t_p - t_n) e^{-\frac{(t' - t)}{\tau}} dt = \\ \tau R_{ss}(\frac{x_1}{v} - t_r + n t_p + t_n - T), \quad (13)$$

assuming that the integration time $t' \gg \tau$. In photorefractive crystals the time constant τ is inversely proportional to the DC intensity level $\tau \propto \frac{\tau_o}{I_{DC}}$, where $I_{DC} = |b_o|^2 + |b_1|^2$. Finally, for flat frequency response broadband signals over the full processor bandwidth we can approximate $R_{ss}(\alpha) \approx \delta(\alpha)$,

$$\tau R_{ss}(\frac{x_1}{v} - t_r + n t_p + t_n - T) \approx \tau \delta(\frac{x_1}{v} - t_r + n t_p + t_n - T). \quad (14)$$

Noting that $\int_0^L |a(z_1 - n D_1)|^2 dz_1 = 1$ is just the sum of the normalized optical power present in the n^{th} fiber

$$\begin{aligned}
E_d^p(x_1, t') &= e^{j\omega_c t'} e^{jkL} b_0 b_1^2 \eta_{AO} \eta_p^2 \times \\
&\quad \sum_{n=1}^N \tilde{s}(t' - n t_p - t_n) \tau \delta\left(\frac{x_1}{v} - t_r + n t_p + t_n - T\right) e^{j[\omega_o(t' - t_r)]} \hat{p}_y \\
&= e^{j\omega_c t'} e^{jkL} \tau b_0 b_1^2 \eta_{AO} \eta_p^2 \tilde{s}\left(t' + \frac{x_1}{v} - t_r - T\right) e^{j[\omega_o(t' - t_r)]} \text{rect}\left[\frac{x_1 - v(t_r + T)}{N v t_p}\right] \hat{p}_y.
\end{aligned} \tag{15}$$

where the rect function defines the spatial region over which we see a diffracted signal due to the sum being only from 1 to N. Note that all the phase factors have been removed in the diffracted signals and over a region defined by the transverse width of the grating envelope we will see a traveling wave version of the signals from the various elements of the phased array propagating at a velocity v independent of the arrival angle of the signal.

As described so far, the readout diffracted beam from the phased array is not separable in angle from the diffracted write beam from the AOD. For our adaptive phased-array processor, it is important to do so. This is accomplished by first vertically deflecting a portion of the light from the phased array fiber feed by a Rochon prism. The polarization of the deflected beam is orthogonal to the undeflected beam, meaning that it has a polarization $\hat{p}_z$ since the polarization of the undeflected beam is $\hat{p}_y$, and the beams are propagating in the x-direction. This is shown in the k-space diagram in Fig. 2. The second beam is tilted vertically in the direction of Bragg degeneracy, allowing for the light to diffract off the original grating. This provides a diffracted beam which is vertically tilted from the reference signal beam from the AOD so that in the Fourier plane they are spatially separated and the AO diffracted beam can be blocked.⁴ A linear polarizer with the transmission axis in the x-direction is used to block the diffracted beam from the AOD (which is polarized in the y-direction), and a diffractive optical element is used to deflect the DC beam from the AO into the same plane as the diffracted beam from the photorefractive crystal. Placing a lens just after the DOE and spatial filtering the beams in the Fourier plane allows us to then to fully separate the diffracted phased array beam from our reference signal. The undiffracted light from the AOD, however, is still allowed to pass so that heterodyne detection can be performed at the TDI CCD. A final lens can then be used to image the diffracted beam, coming from the phased array in the photorefractive crystal, and the undiffracted beam from the AOD, onto the CCD.

Assuming a magnification of the 4f system of M_3 , the total field at the TDI CCD is

$$\begin{aligned}
E_d(x_2, t') &= e^{-j(k_B x_2 - \omega_c t')} e^{jkL} \hat{p}_x \left[\beta_1 \sqrt{1 - \eta_{AO}^2 |\tilde{r}|^2} + \right. \\
&\quad \left. \beta_2 \tau b_0 b_1^2 \eta_{AO} \eta_p^2 \text{rect}\left[\frac{x_1 - v(t_r + T)}{N v t_p}\right] \tilde{s}\left(t' + \frac{x_2}{v} - t_r - T\right) e^{j[\omega_o(t' + \frac{x_2}{v} - t_r - T)]} \right], \tag{16}
\end{aligned}$$

where $x_2 = \frac{x_1}{M_3}$ and β_1 and β_2 are the overall transmission coefficients of the beams through the read-write multiplexing architecture.

The intensity at the detector is given by

Read-Write Multiplexing Technique

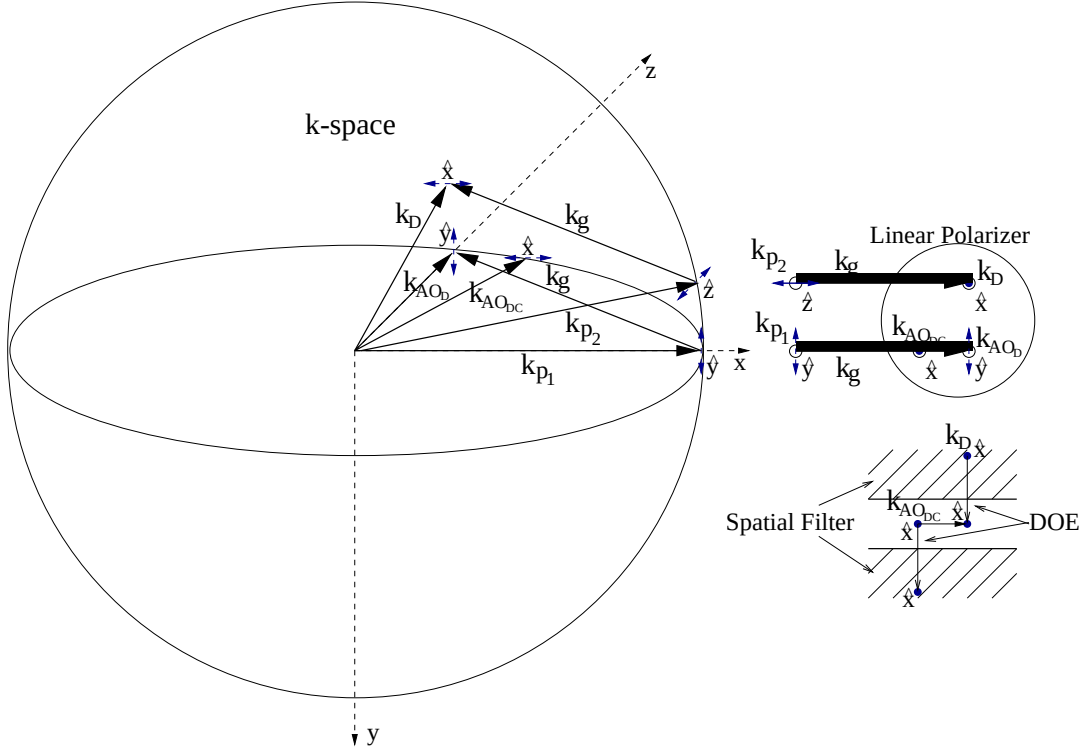


Figure 2: k -space representation of the polarization and angle multiplexed recording and readout geometry for the read-write multiplexing technique used to separate the phased array beam diffracted by the photorefractive crystal from the modulated AOD beam used to write the grating. The inset shows an end on view of the beams in the crystal and the geometry in the Fourier plane of the diffracted beam from the photorefractive crystal and the DC beam from the AOD simultaneously deflected by a vertical diffractive optical element with the same deflection angle as the Rochon.

$$I(x_2, t') = \beta_1^2 (1 - \eta_{AO}^2 |\tilde{r}|^2) + \beta_2^2 \tau^2 b_0^2 b_1^4 \eta_{AO}^2 \eta_p^4 |\tilde{s}(t' + \frac{x_2}{v} - t_r - T)|^2 + \beta_1 \beta_2 \tau b_0 b_1^2 \sqrt{1 - \eta_{AO}^2 |\tilde{r}|^2} \eta_{AO} \eta_p^2 \tilde{s}(t' + \frac{x_2}{v} - t_r - T) e^{j[\omega_o(t' + \frac{x_2}{v} - t_r - T)]} + cc. \quad (17)$$

where we have taken $|rect[\frac{x_1 - v(t_r + T)}{N v t_p}]|^2 = 1$. Note that this interference pattern is moving at a velocity v . Detecting with a synchronously scrolling TDI CCD is done in order to give our final output,

$$o(t) = \int_0^L I(x_2, t') \delta[(t' - t) + \frac{x_2}{v}] dt'. \quad (18)$$

The TDI CCD scrolls at the same rate as the signal since the overall magnification (which scales

the velocity) of the imaging system can be changed with the proper choice of lenses and easily fine tuned by changing the CCD shift rate. The output signal is therefore

$$o(t) = \beta_1^2(1 - \eta_{AO}^2|\tilde{r}|^2) + \beta_2^2\tau^2 b_0^2 b_1^4 \eta_{AO}^2 \eta_p^4 |\tilde{s}(t - t_r - T)|^2 + \beta_1 \beta_2 \tau b_0 b_1^2 \sqrt{1 - \eta_{AO}^2|\tilde{r}|^2} \eta_{AO} \eta_p^2 \tilde{s}(t - t_r - T) e^{j[\omega(t-t_r-T)]} + cc. \quad (19)$$

Our output signal is a coherent sum of N signals from the phased array, with adaptive compensation for all of the corresponding time delays and phase errors. The desired signal is on a temporal carrier at ω_{rf} which allows the signal dependent bias terms to be removed with a bandpass filter. The processed output is the desired beamformed signal from the phased array. This signal can be used as feedback for the adaptive jammer nulling BEAMTAP system or utilized as is since it has achieved the full coherent array gain.

3 Simulation

Figure 3 illustrates a simulation of the BEAMTAP system for the open loop case analyzed in this paper. The array signal is a repetitive far-field broadband spatio-temporal linear chirp spanning from 1.5 GHz to 2.5 GHz. The simulated array has 128 antenna elements and 64 delay taps for a 128×64 adaptive weight matrix. Reference signal $d(t)$ sets up a traveling fringe wave in the AOD which modulates an incident optical beam. This modulated field interferes in the photorefractive crystal with the array field and at the position of maximum correlation strength, corresponding to a minimum time shift between the two field, a grating-let is formed for each antenna element. At steady state, this grating-let diffracts subsequent Bragg matched array fields towards the TDI CCD, where it is accumulated in time with other antenna contributions, producing the output signal $o(t)$.

Notice that as predicted, the overall grating slant angle is directly proportional to the angle of arrival of the array signal. This is because all inherent and random time and phase delays of the array field, determine the corresponding spatial location of each grating-let. On subsequent read out, the shape of the overall grating transforms the diffracted array signal into a coherently propagating wave synchronous with the TDI CCD, yielding a coherent recovery of the array signal with the full coherent antenna gain.

4 Conclusion

We have shown an analytical description of the optical implementation of a photorefractive phased array beamforming system using the BEAMTAP algorithm for a large N -element array. The TTD beamforming operation provides the full coherent array gain, while compensating for inherent and random time delays, as well as any phase distortions which are present in the signal, allowing for detection of both narrowband and broadband signals of interest in both the near-field and far-field regions. Simulation results show that the processor is able to adapt to the

broadband signal of interest at a specific angle of arrival and null out any subsequent jamming signals, allowing the processor to fully adapt to a changing signal environment in real time.

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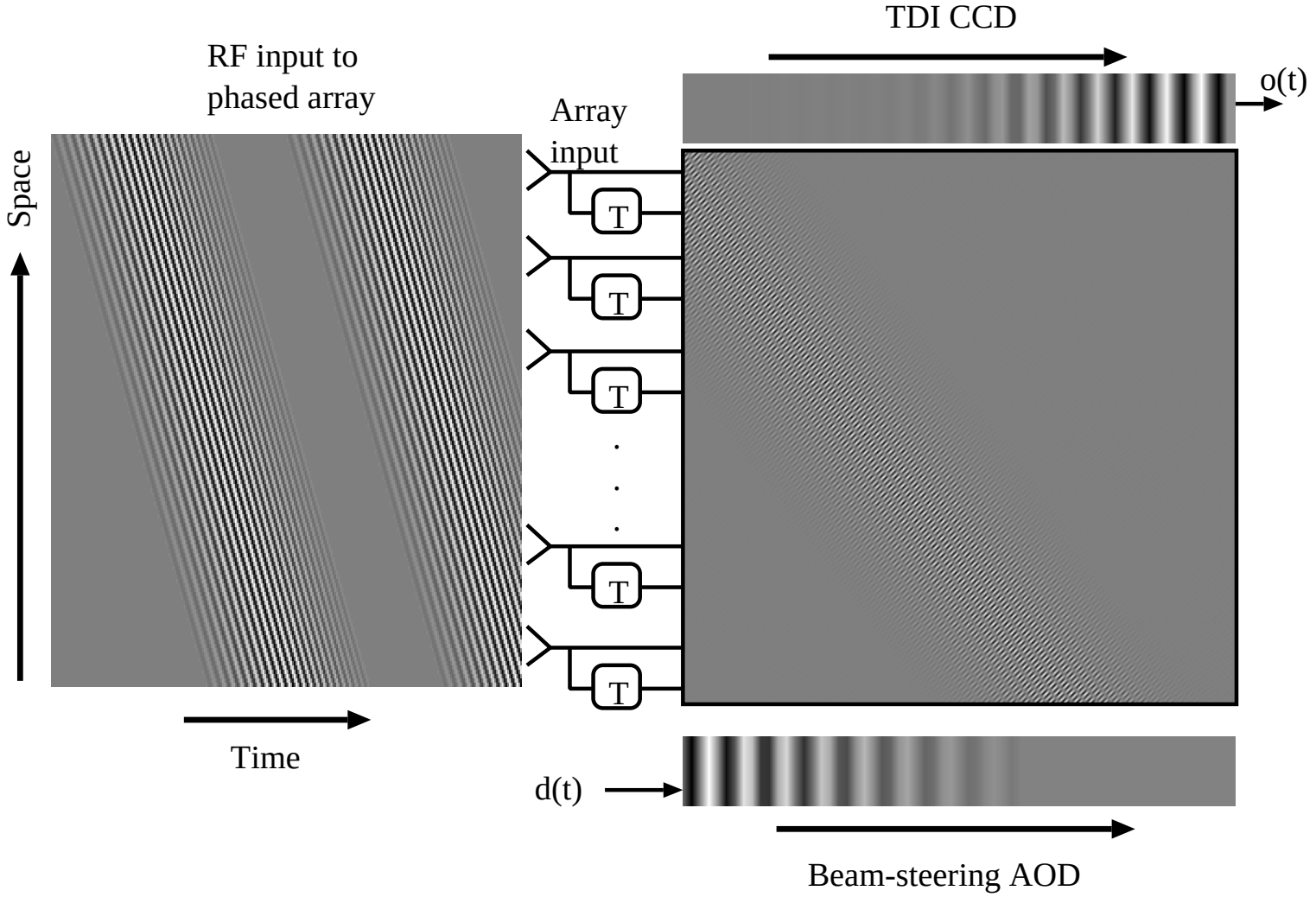


Figure 3: Simulation of BEAMTAP adaptation for a far-field repetitive chirp signal. The space-time incident signal is shown on the left, and two chirp waveforms can be seen. The chirp reference can be seen starting to propagate through the traveling wave modulator on the bottom. A tilted grating slice has built up within the array of weights. The previous chirp incident on the array has diffracted off of this tilted grating and been accumulated in the TDI CCD at the top to reconstruct the desired chirp from the array signals.